Type I planet migration in a magnetized protoplanetary disk

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Introduction

We study the effects of a large-scale, ordered magnetic field in protoplanetary disks on Type I planet migration using a combination of numerical simulations in 2D and 3D and a linear perturbation analysis. Steady-state models of such disks require the inclusion of magnetic diffusivity. To make progress using ideal MHD, we focus on simplified field configurations, involving purely vertical (Bz) and azimuthal (Bl) field components and a combination of the two. Additionally, we study the case of a wind-driving disk, in which a magnetic torque ~ Bz dBz/dz (where Bz and dBz/dz are the equatorial vertical and azimuthal components of the magnetic fields), induces vertical angular momentum transport. For each of the models we calculate the locations of the relevant resonances and of the turning points, which delineate the propellant regions of the MHD waves that transport angular momentum from the planet to the disk. We use both numerical and semianalytical methods to evaluate the cumulative back torque acting on the planet. We conclude that, under realistic circumstances, a large-scale magnetic field can slow down inward migration that characterizes the underlying unmagnetized disk — by up to a factor of ~ 2 when the magnetic pressure approaches the thermal pressure — but it cannot reverse it. A previous inference that a pure-Bz field whose amplitude decreases fast enough with radius leads outward migration applies only in 2D. For a wind-driving disk configuration, we find the presence of a subdominant Bz component whose amplitude increases from zero at z = 0 has little effect on the torque when acting alone, but in conjunction with a Bz component it gives rise to a strong torque that speeds up the inward migration of the planet.

Linear Perturbation Analysis

Basic Equations and Disk Response

Dynamics of disk are governed by momentum, conservation of mass, and induction equations under ideal MHD:

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla P + \rho \mathbf{g} + \nabla \cdot \mathbf{J} \times \mathbf{B} \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \nabla \times \mathbf{E} \]

where \( \rho \) is the mass density, \( \mathbf{v} \) the gravitational potential, \( \mathbf{v} \) the velocity field, \( \mathbf{B} \) the magnetic field, and \( \mathbf{R} \) the Larmor force density.

We calculate the disk response to the planet (perturbation potential \( \psi \)) by perturbing the equations in all 3 directions and time in Fourier space (with Fourier labels \( m \) and \( k \) for the azimuthal and vertical directions) and then linearizing. Details in Bans, Königl & Uribe (2014). Linearized equations yield a differential equation for the radial Lagrangean displacement that can be solved at each azimuthal mode by using high-order adaptive step Runge-Kutta methods with 16-bit precision.

Resonances

Pure Resonances: where the density perturbation is divergent (i.e., \( \lambda < 0 \)).

Two most basic magnetic resonances where the perturbation frequency is near an MHD-Oort resonance, and Alfvén resonances where the constant perturbation frequency is equal to that of a sheet Alfvén wave traveling in the vertical direction.

Turning Points

Where the differential equation above goes from wave-like solutions to evanescent are the so-called turning points (details for solving for the turning points are presented in Bans, Königl & Uribe (2014)).

Vertical Fields and Toy Wind Model (2D and 3D)

Black solid line shows the planet’s path for a purely vertical field configuration as a function of the azimuthal coordinate \( \phi \). The vertical wind model, \( \phi \)-dependent, is fixed at \( \phi = 0 \) and the horizontal wind model, \( \phi \)-independent, is fixed at \( \phi = 0 \). The vertical field model, \( \phi \)-dependent, is fixed at \( \phi = 0 \), while the horizontal field model, \( \phi \)-independent, is fixed at \( \phi = 0 \).

Azimuthal and Vertical Fields (2D):

Toy wind model is a simplified version of the standard wind model. The wind is a function of each radial point and its vertical position. The wind is a function of the vertical position and the vertical position of the planet. We assume that the planet’s path is determined by the vertical wind model.

Numerical MHD simulations

The protoplanetary disk

The disk is modeled as a sub-Kuiper belt mean-field in two and three dimensions. The geometry is cylindrical (r,\( \phi \), z). The gas torque on the radial component of the spiral density, \( \psi \), and it is unsaturated. The disk is thrusted by a strong and uniform magnetic field of the general form

\[ \psi = \psi_{0} (r, \phi, z) \]

Planet-disk interaction

Planet in a disk with a vertical field. The planet is located at (r,\( \phi \),z) and the disk is located at (r,\( \phi \),z). The planet is excited by the uncalculated force acting on the planet, \( F \), and the disk is excited by the uncalculated force acting on the disk, \( F \).

Radial migration

This case of migration of the planet (\( \psi \)) is estimated by calculating the gravitational specific torque

\[ \Gamma = \frac{C_{\psi}}{P_{\psi}} \]

Acknowledgments

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References